DEPARTMENT OF STATISTICS				*CLASS: I M.Sc. Statistics*				
Sem	**Course Type**	**Course Code**	**Course Title**	**Credits**	**Contact Hours/week**	**CIA**	**Ext**	**Total**
II	Major Core -4	21P2SMC4	Measure and Probability Theory	4	5	25	75	100

Nature of Course				
Knowledge and skill	✓		Employability oriented	
Skill oriented			Entrepreneurship oriented	

Course Objectives:

1. To gain the knowledge on Set theory, limits, probability and inequalities
2. To connect between probability theory and other branches of mathematics in particular measure theory, complex and functional analysis

Unit	Description	Hours	K-level	CLO(s)
I	Limit Superior, Limit inferior and limit of a sequence of sets – Field and Sigma field - Monotone class – Functions and inverse functions – Borel field – Countable and finitely additive set function – Measurable space – Measure space –Measure, Properties of Measure. Lebesgue – Steiltjes measure – Lebesgue measure. Measurable function, Simple function. Concept of almost everywhere. Approximation theorem (statement only).	17	K2	1
II	Measure Integral – Properties – Monotone convergence theorem – Fatou's lemma –Dominated convergence theorem (statement only) – Absolute continuity of two measures –Radon-Nykodym theorem (statement only). Product sets and Fubini's theorem (statement only).	15	K3	2
III	Probability measure – Random variable —distribution function – Expectation – inequalities – Jensen, Basic, Cauchy-Schwartz. Conditional probability - Conditional expectation – Independence of random variables – Borel-Cantelli Lemma – Kolmogorov Zero – one law.	16	K4	3
IV	Convergence of random variables – almost sure, in law, in probability, in rth mean and their interrelations – Characteristic function – Inversion formula, Convergence of distribution functions and characteristic functions –Helly-Bray theorem – Continuity theorem, Uniqueness theorem.	13	K4	4
V	Law of large numbers – Weak law of large numbers – Kolmogorov's strong law of large numbers –Glivenko – Cantelli Theorem – Central limit theorems – de-Moivre's – Laplace Theorem, Lindeberg–Cramer - Levy theorem, Liapounov's theorem.	14	K4	5

Books for Reference:

1. De Barra, G. (1987) Measure Theory and Integration, Wiley Eastern, New Delhi.
2. Tucker, H.G. (1967) A Graduate Course in Probability, Academic Press, New York.
3. Munroe, M.E. (1965) Measure and Integration, Addison & Wesley, New York.
4. Bhat B. R, (2014), Modern Probability Theory (Fourth Edition), New Age International, New Delhi (Reprint 2015).
5. Ash, B.R, (1972), Real Analysis and Probability, Academic Press, New York.
6. Billingsley P, (2012), Probability and Measure (Third Edition), John Wiley & Sons, New York.
7. Chow, Y.S. and Teicher, H, (2012), Probability Theory; Independence, Interchange ability, Martingales (Second Edition). Springer Limited.
8. Feller, W. (1972), An Introduction to Probability Theory and Its Applications, Volume II, John Wiley & Sons, New York. (Reprint, 2008).
9. Loe ̓ve, M. (1978), Probability Theory (Fourth Edition), Springer-Verlag, New York.
10. Rana, I.K. (2005), An Introduction to Measure and Integration (Second Edition), Morgan & Claypool.
11. Rohatgi, V.K. and Saleh, A.K.Md.E. (2011), An Introduction to Probability and Statistics (Second Edition). John Wiley & Sons, New York.
12. Ross, S.M (2010). A First Course in Probability.Pearson Prentice Hall.

Web references:

1. Measure & Probability: https://nptel.ac.in/courses/111/101/111101100/#
2. Law of Large Numbers: https://www.youtube.com/watch?v=VpuN8vCQ--M
3. Central Limit Theorem: https://www.youtube.com/watch?v=JNm3M9cqWyc
4. https://www.youtube.com/watch?v=YAlJCEDH2uY
5. Inequalities in Probability: https://www.youtube.com/watch?v=nrDkb2MAwSA
6. https://www.youtube.com/watch?v=DWsDqKlW7Z4

Rationale for Nature of the course

This course introduces measure theory in a rigorous way and explores some applications to probability theory. Both of these are core mathematical disciplines. In addition, the knowledge of probability theory provided in this course is essential basis for further courses in mathematical finance.

Activities having direct bearing on Skill development / Employability / Entrepreneurship

Exercise problems relate to probability or chance based given on realtime situations

Pedagogy

Chalk and Talk, PPT, Seminar, Interaction, Problem solving.

Lecture Schedule

Unit	Topics	Hours	Mode
I	Limit Superior, Limit inferior and limit of a sequence of sets	**3**	PPT, Chalk and Talk and Assignments
	Field and Sigma field	**2**	
	Monotone class – Functions and inverse functions	**3**	
	Borel field	**1**	
	Countable and finitely additive set function – Measurable space – Measure space –Measure, Properties of Measure	**2**	
	Lebesgue – Steiltjes measure – Lebesgue measure	**2**	
	Measurable function, Simple function. Concept of almost everywhere	**2**	
	Approximation theorem (statement only).	**2**	
II	Measure Integral – Properties	**3**	PPT, Chalk and Talk and Assignments
	Monotone convergence theorem – Fatou's lemma	**4**	
	Dominated convergence theorem (statement only)	**3**	
	Absolute continuity of two measures	**2**	
	Radon-Nykodym theorem (statement only). Product sets and Fubini's theorem (statement only).	**3**	
III	Probability measure	**2**	PPT, Chalk and Talk, Assignments and seminar
	Random variable –distribution function Expectation	**3**	
	inequalities – Jensen, Basic, Cauchy-Schwartz.	**3**	
	Conditional probability - Conditional expectation	**4**	
	Independence of random variables	**2**	
	Borel-Cantelli Lemma – Kolmogorov Zero – one law	**2**	
IV	Convergence of random variables – almost sure, in law, in probability, in rth mean and their interrelations —	**4**	PPT, Chalk and Talk, Assignments and seminar
	Characteristic function – Inversion formula	**2**	
	Convergence of distribution functions and characteristic functions	**3**	
	Helly-Bray theorem	**2**	
	Continuity theorem, Uniqueness theorem. State Integrated Board of Studies – Statistics PG	**2**	
V	Law of large numbers	**2**	PPT, Chalk and Talk, Assignments and seminar
	Kolmogorov's strong law of large numbers-Glivenko	**2**	
	Cantelli Theorem – Central limit theorems de-Moivre's	**3**	
	Weak law of large numbers - Laplace Theorem	**3**	
	Lindeberg–Cramer - Levy theorem,	**2**	
	Liapounov's theorem.	**2**	

Course Learning Outcomes

On the successful completion of the course, students attain

CLO's	Course Learning Outcomes	Knowledge Level
CLO-1	The knowledge on function, limit and set theory	Up to K2
CLO-2	The knowledge on Measure integral and to apply convergence theorem	Up to K3
CLO-3	To derive the inequalities and apply the probability in distributions	Up to K3
CLO-4	To identify the convergence of sequences/situations	Up to K4
CLO-5	To apply the law of large numbers and central limit theorem in the real time situations	Up to K4

MAPPING CLOs WITH PSOs

#	PSO1	PSO2	PSO3	PSO4	PSO5	PSO6	PSO7
CLO-1	2	2	3	2		2	2
CLO-2	1	2	1	2		1	2
CLO-3	1	3	2	2		2	2
CLO-4	1	2	2	2		1	2
CLO-5	1	1	2	2	2	2	2

Advance application – 3; Intermediate level – 2; Basic level – 1

CIA - I - BLUE PRINT

CLO's	K-Level	Section – A Short Answer		Section B Either / Or type		Section C Open Choice	
		No. of Questions	K Level	No. of Questions	K Level	No. of Questions	K level
CLO 1	Up to K2	3	K1, K1, K2	2	K1, K2	1	K2
CLO 2	Up to K3	2	K2, K3	2	K2 , K3	2	K3, K3
No. of questions to be asked		5		4		3	
No. of question to be answer		5		2		2	
Marks for each question		2		5		10	
Total marks for each section		10		10		20	

CIA-I :: Distribution of Section-wise Marks with K levels

K Levels	Section A (No choice)	Section B (Either/ or)	Section C (Open choice)	Total marks	% of marks without choice	Consolidated
K1	4	5	-	9	15.00	55%
K2	4	10	10	24	40.00	
K3	2	5	20	27	45.00	45%
K4	-	-	-	-	-	-
K5	-	-	-	-	-	-
Total Marks	10	20	30	60	100	100%

CIA-II – Blue Print

Units	CLOs	K- Level	Section A		Section B		Section C	
			Short Answers		(Either/or Choice)		(Open Choice)	
			No. of Questions	K- Level	No. of Questions	K- Level	No. of Questions	K- Level
1	CLO 3	Up to K3	2	K1, K2	2	K2, K3	1	K3
2	CLO 4	Up to K4	3	K1, K2,K3	2	K3, K4	2	K2, K4
No. of Questions to be asked			5		4		3	
No. of Questions to be answered			5		2		2	
Marks for each question			2		5		10	
Total Marks for each section			10		10		20	

CIA-II :: Distribution of section wise marks with K levels.

K Levels	Section B (Short Answers)	Section C (Either/ or)	Section D (Open choice)	Total marks	% of marks without choice	Consolidated
K1	4	-	-	4	6.67	38.33%
K2	4	5	10	19	31.66	
K3	2	10	10	22	36.67	36.67%
K4	-	5	10	15	25	25%
K5	-	-	-	-	-	-
Total marks	10	20	30	60	100	100

Summative Examination– Blue Print

S.No	CLOs	K-level	Section A		Section B		Section C (Either/or Choice)	Section D (Open Choice)	Total
			MCQs		Short Answers				
			No. of Questions	K-Level	No. of Questions	K-Level			
1	CLO 1	Up to K2	2	K1 & K1	1	K1	2(K1& K1)	1(K2)	
2	CLO 2	Up to K3	2	K2& K3	1	K1	2(K2 & K2)	1(K3)	
3	CLO 3	Up to K3	2	K2 & K3	1	K2	2(K3 & K3)	1(K3)	
4	CLO 4	Up to K4	2	K3 & K4	1	K2	2(K4& K4)	1(K4)	
5	CLO 5	Up to K4	2	K3& K4	1	K3	2(K4& K4)	1(K4)	
No. of Questions to be asked			10		5		10	5	30
No. of Questions to be answered			10		5		5	3	23
Marks for each question			1		2		5	10	
Total Marks for each Section			10		10		25	30	75

Distribution of sectionwise marks with K levels for Summative Examination

K - Level	Section A (MCQ)	Section B (Short Answer Questions)	Section C (Either / Or Choice)	Section D (Open Chioce)	Total Marks	% of (Marks without Choice)	Consolidated
K1	2	4	10	-	16	13.33	33.33%
K2	2	2	10	10	24	20.00	
K3	4	4	10	20	38	31.67	31.67%
K4	2	-	20	20	42	35	35%
K5	-	-	-	-	-	-	-
Total	10	10	50	50	120	100	100

Course Designers:

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